Income Distribution, Vertical Differentiation, and the Quantity Competition

Zhuang Miao
McGill University

Advisor: Ngo Van Long
McGill University

Abstract

The paper analyzes the effects of the change of the income distribution on the equilibrium outcomes in the duopoly-quality model with quantity competition. The analysis results show that with zero quality-cost and an income inequality not too high, then both firms always choose the highest quality level. If the quality-cost is convex, then the average quality level will decrease and the vertical differentiation level will increase in the income inequality. These results are different from the Yurko (2011), who made a similar analysis under the quality-price competition model. Another contribution of the paper is that it gives the sufficient conditions for the single firm to choose multiple levels of the quality, i.e. the quality-cost function is convex, vertical differentiation is large enough, and the marginal cost is not too high.

JEL classification: L10 L13 D22
Key words: Quality, Price, Vertical Differentiation, Cournot Competition, Income Distribution

1 Introduction

One important topic in the economics of industrial organization is to study the impact of income inequality on prices and quality levels. The recent empirical studies also show that the income distribution does affect the equilibrium variables of the market. Frankel (2001) studied the effect of income inequality on retail prices, using US data. He found that an increase in the presence of lower-middle income households, relative to poor or upper income households, is associated with lower prices. In other studies, Krueger and Perri (2006) and Jappelli and Pistaferri (2010) found that consumption patterns change in response to changes in income inequality. Benassi, Chirco, and Colombo (2006) analyzed the effects of the income distribution on the quality of the products under a quality-price competition model. They found that lower income inequality will lead to lower product quality. In another paper, Yurko (2011) studied the change of the products' average quality provided by the firms in response to the variety of the income inequality. Yurko (2011) also reached the similar conclusion that high average quality is associated with high level of income inequality. The bulk of the literature on this subject assumes that firms compete in prices after they have selected their quality levels. Furthermore, most authors assume that firms do not incur additional costs when they increase their quality levels. However, the results may alter under the Cournot model because the main factor that leads to the results in the Bertrand model is the high intensity of the competition. The intensity of the competition under the Cournot model is much less alleviated than the Bertrand model. Considering the Cournot competition exists in many industries, it is necessary to study the issue under the Cournot model. The main purpose of the paper is to study the effects of the income inequality on the outcomes under the assumption that firms set quantities (rather than prices) after they have chosen their quality levels. The contributions of this paper include: firstly, it is the first one to analyze the correlation between the product's quality and the income inequality under the Cournot model; secondly, it is the first paper to study the within-firm vertical differentiation, i.e. the single firm chooses multiple quality levels of the products.

This paper shows that if the cost of quality-improvement is zero and the income inequality is not too high, then both firms will choose the highest quality level. With the quadratic form of quality-cost and the lognormal form of income distributions, when the income inequality enlarges, the vertical differentiation level increases, but the average quality decreases. These results, obtained under the assumption of quantity competition rather than price competition, are different from those of the Yurko (2011), which assumes the quality-Bertrand model. In Yurko (2011), no matter what the cost structure is, the average quality level is always increasing and the vertical differentiation is decreasing in the income inequality. The paper also discusses the situation when the single firm is allowed to choose more than one level of the quality. This assumption is more in line with reality, though in the existing literature the typical assumption is that each firm can only choose one quality level. This paper shows that in the monopolistic market the firm will vertically differentiate its products if the cost is non-linear and increasing in both the quality level and the quantity of the products. In the case of a duopoly, the necessary condition for the firms to make within-firm vertical differentiation is that the cost function is increasing in quantity and convex in the quality-level.

2 Literature review

Benassi, Chirco, and Colombo (2006) analyzed the effects of the income distribution on the vertical differentiation and the market structure under a quality-Bertrand competition model. They assumed zero cost on quality investment and the consumers' income follow the trapezoid distribution. Under this construction, they reach the result that a more centralized income distribution will expand the vertical differentiation. The intuition of the result is that the higher income concentration stimulates the market competition and this effect can only be damped by the enlarging of the vertical differentiation. In another paper, Yurko (2011) analyzes the change of the average quality of the products in response to increases in the income inequality under the assumptions of free entry and Bertrand competition. She found that when the income become more unequal, more firms will enter the market and then the market competition will become more intensive. Then the average quality of the products will increase because firms compete for the shrinking share of higher-income consumers. Although Benassi, Chirco, and Colombo (2006) and Yurko (2011) obtained the same conclusion about the change of quality in response to the change of the income distribution, their underlying mechanisms are different. In Benassi, Chirco, and Colombo (2006) the market competition takes place in the middle-income group, meanwhile in Yurko (2011) the competition occurs in the market serving the high income costumers. When the income inequality becomes larger, the market competition becomes less fierce in Benassi, Chirco, and Colombo (2006) but becomes more intensive in Yurko (2011). In Benassi, Chirco, and Colombo (2006) the moderated market competition allows the low quality firm to narrow the quality gap relative to the high quality firm. In contrast in Yurko (2011), the enhanced competition induces the firms to raise the quality level in order to attract the wealthier consumers.

Several papers have studied quality choice when firms compete in quantity. However, none of them analyzed the effects of the change of the market characteristics on the market outcomes. Bonanno (1986) discussed the properties of the vertical differentiation model with both the Bertrand and Cournot types of competition. He reached the conclusion that there exists subgame-perfect Nash equilibrium under the quality-Cournot competition, and both firms will choose the highest level of quality. He also discussed the results when quality improvement involves a cost. Bonanno (1986) assumes that each firm must choose a quality level greater than a threshold c which is strictly positive. He claimed that if the cost is high then one firm will choose the lowest level of quality meanwhile the other firm stays with the highest level of the quality. The difference of the cost function between Bonanno (1986) and our setting is that in our paper the quality is chosen from the range starting from zero and the cost is increasing in the quality-level. In our analysis, if the firm chooses zero quality, which means the firm exits the market. Displacing the lower bound of the quality range makes the setting more closed to the reality. Another significant difference between Bonanno's model and our model is that in our model, the range of consumer valuation of quality changes as the support of the income distribution changes. In another paper, Frascatore (2002) explored the situation where the cost of the quality-improvement depends on the quality levels chosen by both firms. Frascatore argues that the cost for a firm of achieving a quality level depends on the quality level of the other firm. Thus, if the quality level of firm 1 is S_1 and the quality level of firm 2 is S_2, Frascatore assumes that the cost of quality is a function of the summation, i.e. $S = S_1 + S_2$. With this assumption and under the Cournot model, he found that one firm will choose a positive level of quality but the other firm will choose zero quality. In another paper, Motta (1993) compared the results between the Bertrand and Cournot models with assuming a convex cost function for the quality-improvement. He found that with such type of cost structure, the firms will differentiate the quality in both the Bertrand and the Cournot model. Motta (1993) also compared the social welfare under the both models, and reached the conclusion that the social welfare associated with the Bertrand model is always higher than the Cournot model. Our paper also compares the results from both models. Contrary to the results of Motta (1993), our model shows that if the marginal cost of the quality is constant and high enough, then the social welfare of the Bertrand model will be lower than the Cournot model. Unlike the models that I surveyed above, which assume that a consumer buys at most one unit of the differentiated good, Symeonidis (2003) assumes that a consumer can choose the number of units she buys from each duopolist. The utility function is quadratic in quantities. He reached the conclusion that when the R&D spillovers is high enough or the vertical differentiation is low enough, then the welfare in the Bertrand model will be lower than the Cournot model. Both Symeonidis (2003) and our analysis show that under certain conditions, the Bertrand model is not more efficient than the Cournot model.

The other key papers which analyze the quality-Cournot model include Lambertini and Tampieri (2012a), Lambertini and Tampieri (2012b), Aoki (2003), Nguyen, Sgro, and Nabin (2014), Andaluz (2010), and Hergueraa, Kujalb , Petrakisc (2000). Lambertini and Tampieri (2012a) explored the behaviors of the firms in providing the environmental friendly products and reached the conclusion that the Porter-type result holds in this model. The Porter-type result means that the social welfare will increase with government intervention in the market. In another paper, Lambertini and Tampieri (2012b) assumed that firms participate in Stackelberg competition in the stage of choosing qualities and make simultaneous quantity competition. The analytical result shows that the low quality firm prefers to be the leader in choosing the quality. Aoki (2003) compared the equilibrium quality levels between the sequential and simultaneous types of games under both the Bertrand and Cournot models. He showed that under the Bertrand model the equilibrium qualities will be lower in the sequential game than the simultaneous game. Meanwhile under the Cournot model, the high quality will be higher but the low quality will be lower in the sequential game than the simultaneous game. Nguyen, Sgro, and Nabin (2014) studied the welfare effects of the endogenous choice of the cost structure in the quality-improvement process under both the Bertrand and Cournot models. They assumed that the domestic firms could choose two methods to improve the quality levels: first one is making quality-investment by themselves with a convex cost function; the second one is to buy license from the foreign firm with the lump-sum cost. They found that licensing raises domestic welfare, and the welfare is higher in Bertrand than in Cournot competition regardless of the cost structure. Andaluz (2010) studied the stability of the collusion in both the quality-Bertrand and the quality-Cournot models. He found that the effect of the vertical differentiation on the sustainability of the collusion is unclear, but the collusion is more stable in the Bertrand model than the Cournot model. Hergueraa, Kujalb , and Petrakisc (2000) studied the effects of the quota restrictions on the quality choice by the export firms. They found that if the import country imposes quota restrictions, then the low quality firm would increase the quality level and the high quality firm would keep the same quality level as before the imposition of the quantity restrictions.

Another important contribution of the paper is that it discusses the conditions for the change of the market structures between the covered and uncovered under the Cournot model. It also discusses the conditions for the existence of the sub-perfect Nash equilibrium under the Cournot model. The discussions provide important fundamentals to the studies of the relevant issues. The first paper which discussed the boundaries of the market structures is Wauthy (1996). The author discussed the conditions for the holding of each type of market structures (uncovered market, covered market with the corner solution, and with the interior solution) under a quality-Bertrand model and assuming zero quality-cost. In our analysis, we extend the discussion under the quality-Cournot competition model.

The last contribution of the paper is that it explores the cost structures and other conditions that gives incentives for a single firm to provide the multiple levels of quality in both the monopolistic and duopolistic markets. This issue was rarely studied before. We found three similar literature on this topic, i.e. Mussa and Rosen (1978), Crawford and Shum (2007), and Garella and Lambertini (1999). Mussa and Rosen (1978) and Crawford and Shum (2007) discussed the multiple choices of the quality levels by the monopoly-firm. Lambertini (1999) considered a vertically differentiated duopoly where product quality is assumed as the combination of the good and bad characteristics. Consumers obtain positive utility from the good characteristics and negative utility form the bad characteristics. They reached the results that firms would differentiate the good characteristics but chose the same level of the bad characteristics. In our study, the single firm could provide the products with multiple levels of quality and the market structures are either monopolistic or duopolistic. The analytical results show that, the firm needs to consider the substitute and the cost effects when deciding the provision of the product's quality.

The paper is constructed as followings: section 3 provides the basic models to discuss the effects of the change of income distribution on the market outcomes; the first part of the section 3 explores the case with zero quality-cost, and the second part of the section studies the case with a quadratic quality-cost function; section 4 makes some extensions based on the fundamental models in section 3, i.e. compare the social welfare between the Cournot and Bertrand models, and investigates the conditions for the single firm to choose multiple levels of quality.

3 Theoretical Models

Our model adopts the traditional formulation of vertical product differentiation. There are two firms, i = 1, 2, producing the same type product, and choosing the quality of the product within the range [0, $\bar{S}$], i.e. $S \in$[0, $\bar{S}$]. The firms make competition in two stages. In the first stage, both of them choose the quality level simultaneously, and then in the second stage the firms engage in the price or quantity competition. Assume the production cost in the second stage is zero and the cost of improving the quality of the products in the first stage is denoted by $C(S)$. The quality, quantity and price chosen by firm i are denoted as S_i, X_i and P_i respectively, where i =1,2. Without loss of generality, we assume $S_1 \leq S_2$. The utility function for consumer t is $U_t = \theta_t S_i - P_i$, where θ_t indexes the preference level of the consumer towards the product's quality. We assume that each consumer can purchase no more than one unit of the product. Another convenient assumption is that the consumer with higher income would have higher level of the preference. This assumption is reasonable, because the utility function $V_t = S_i - \frac{P_i}{\theta_t}$ is equivalent to U_t under the settings of the game. We may suppose that given purchasing the same product, the expenditures will be the same, but the rich people will lose less utility from the expenditures than the poor people. In this case, we can index the consumer's income as θ. To simplify the analysis, the income distribution is defined in the range$[0, \bar{\theta}]$ and the CDF, PDF are denoted as $F(\theta)$ and $f(\theta)$ respectively. The consumer with income θ_t will choose to buy the product i rather than buying nothing if $U_t(\theta_t, P_i) > 0$, and he will buy the product j rather than i if $U_t(\theta_t, P_i) > U_t(\theta_t, P_j)$. Denote the type of consumer who is indifferent between choosing low quality product and nothing as θ_L, the type who is indifferent between choosing high and low quality product as θ_I, and the type who is indifferent between choosing the high quality product and nothing as θ_H. Using the utility function specified above, we can obtain the following results: $\theta_L = \frac{P_1}{S_1}$, $\theta_H = \frac{P_2}{S_2}$, and $\theta_I = \frac{P_2 - P_1}{S_2 - S_1}$. Then we further get the following relation: $\theta_I \geq \theta_H \geq \theta_L$. In this case, the consumer whose income is higher than θ_I will buy the high quality products, and the consumers between the type θ_L and θ_I will buy the low quality products. The consumers whose income is lower than θ_L will choose to buy nothing. Figure 1 describes the market for each type of the product. Following Yurko (2011), we study the issue with two cases, the case with zero quality-improvement cost and the case with the quadratic form of the quality cost, i.e. $C(S) = c \cdot S^2$. We assume the production cost in the second stage is zero in both cases.

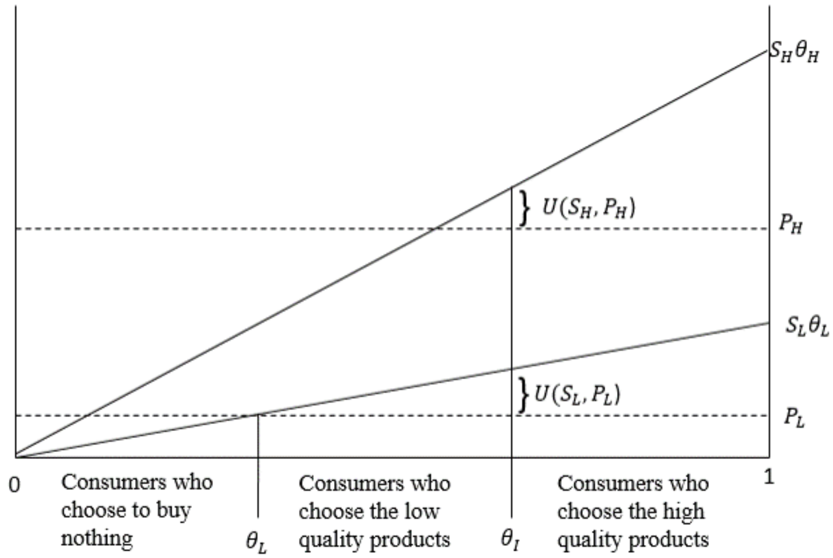

Figure 1. The markets for the high and low quality products

3.1 Income inequality and market outcomes with zero cost

3.1.1 Basic results with uniform type of income distribution

To simplify the analysis, let us at first consider the case of a uniform distribution with the support being the range $\theta \sim U[\theta^-, \theta^+]$, and the quality-cost is zero. In this case the PDF of the θ is $f(\theta) = \frac{1}{\theta^+ - \theta^-}$. As θ is non-negative, so we need the lower bound of the distribution is non-negative, i.e. $\theta^- \geq 0$.

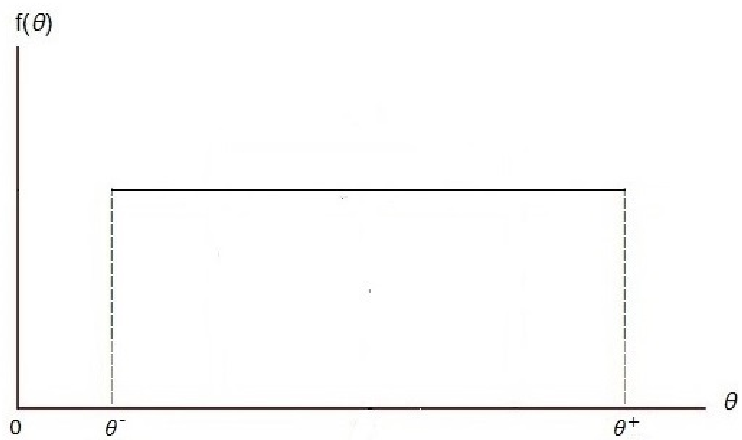

Figure 2. Income distribution with the uniform type

The competition follows two stages. At the first stage, the firms choose their own quality level of the products, and at the second stage, the firms engage in quantity or price competition given the quality chosen in the first stage. We assume for the moment that in equilibrium, the market is uncovered, meaning there are consumers that do not buy the product. Denote the type of consumer who is indifferent between purchasing the low quality product and nothing as θ_L, i.e. $\theta_L S_1 - P_1 = 0$, then $\theta_L = \frac{P_1}{S_1}$. Denote the type of consumer who is indifferent between purchasing product 1 and 2 as θ_I , i.e. $\theta_I S_1 - P_1 = \theta_I S_2 - P_2$, then $\theta_I = \frac{P_2 - P_1}{S_2 - S_1}$. In this case we can get the demand functions of firm 1 and 2 as the followings:

$$\begin{cases} D_1 = & \int_{\theta_L}^{\theta_I} \frac{1}{\theta^+ - \theta^-} d\theta \\ D_2 = & \int_{\theta_I}^{\mu+\delta} \frac{1}{\theta^+ - \theta^-} d\theta \end{cases} \tag{1}$$

where$\theta_L = \frac{P_1}{S_1}$ and $\theta_I = \frac{P_2 - P_1}{S_1 - S_2}$.

Then we get the demand functions for firm 1 and 2 as:

$$\begin{cases} X_1 = & \frac{1}{\theta^+ - \theta^-}\left(\frac{P_2 - P_1}{S_2 - S_1} - \frac{P_1}{S_1}\right) \\ X_2 = & \frac{1}{\theta^+ - \theta^-}\left[\theta^+ - \frac{P_2 - P_1}{S_2 - S_1}\right] \end{cases} \tag{2}$$

Re-ranging the formula (2), we further get the inverse demand functions.

$$\begin{cases} P_1 = & [\theta^+ - (\theta^+ - \theta^-) \cdot (X_1 + X_2)]S_1 \\ P_2 = & [\theta^+ - (\theta^+ - \theta^-) \cdot X_2]S_2 - (\theta^+ - \theta^-)S_1 X_1 \end{cases} \tag{3}$$

The formula (3) can be seen as an example of the inverse demand function of the differentiated Cournot model specified by Dixit (1979) and Singh and Vives (1984), i.e. $P_1 = A_1 - \alpha_1 X_1 - \alpha_2 X_2$ and $P_2 = A_2 - \beta_1 X_1 - \beta_2 X_2$. As the competition follows two stages, so we need to solve the equilibrium quantities given the product's quality in the second stage, and then solve the optimal qualities in the first stage. Assume the cost of the production and quality-improving processes is zero, then the profit function for each firm can be specified as:

$$\begin{cases} \pi_1 = & X_1[\theta^+ - (\theta^+ - \theta^-) \cdot (X_1 + X_2)]S_1 \\ \pi_2 = & X_2\{[\theta^+ - (\theta^+ - \theta^-) \cdot X_2]S_2 - (\theta^+ - \theta^-) \cdot S_1 X_1\} \\ P_1 \geq & S_1 \theta^- \end{cases} \tag{4}$$

The inequality condition in (4) guarantees that the consumer who is indifferent between choosing low quality products and nothing is higher than the lower bound. Take first order conditions of π_1 and π_2 with respect to X_1 and X_2 respectively, we get:

$$\begin{cases} \frac{\partial \pi_1}{\partial X_1} = & \theta^+ S_1 - 2(\theta^+ - \theta^-) \cdot S_1 X_1 - (\theta^+ - \theta^-) \cdot S_1 X_2 = 0 \\ \frac{\partial \pi_2}{\partial X_2} = & \theta^+ S_2 - 2(\theta^+ - \theta^-) \cdot S_2 X_2 - (\theta^+ - \theta^-) \cdot S_1 X_1 = 0 \end{cases} \tag{5}$$

As the second order conditions of the formula in (5) are all negative, i.e. $\frac{\partial^2 \pi_1}{\partial X_1^2} = -2(\theta^+ - \theta^-)S_1 < 0$ and $\frac{\partial^2 \pi_2}{\partial X_2^2} = -2(\theta^+ - \theta^-)S_2 < 0$, so the optimal quantities are solved as the followings:

$$\begin{cases} X_1^* = & \frac{\theta^+}{\theta^+ - \theta^-} \cdot \frac{S_2}{4S_2 - S_1} \\ X_2^* = & \frac{\theta^+}{\theta^+ - \theta^-} \cdot \frac{2S_2 - S_1}{4S_2 - S_1} \end{cases} \tag{6}$$

Further, we can obtain the equilibrium prices as:

$$\begin{cases} P_1 = & \theta^+ \cdot \frac{S_1 S_2}{4S_2 - S_1} \\ P_2 = & \theta^+ \cdot \frac{2(S_2 - S_1)S_2}{4S_2 - S_1} \end{cases} \tag{7}$$

Then we can write out the profits functions for both firms in the first stage in terms of $S_1$and S_2:

$$\begin{cases} V_1 = & \frac{\theta^{+2}}{\theta^+ - \theta^-} \cdot \frac{S_1 S_2^2}{(4S_2 - S_1)^2} \\ V_2 = & \frac{\theta^{+2}}{\theta^+ - \theta^-} \cdot \frac{S_2 (2S_2 - S_1)^2}{(4S_2 - S_1)^2} \end{cases} \tag{8}$$

Take first order conditions of the formula in the set (8) with respect to S_1 and S_2 respectively, we get:

$$\begin{cases} \frac{\partial V_1}{\partial S_1} = & \frac{\theta^{+2}}{\theta^+ - \theta^-} \cdot \frac{S_2^2 (4S_2 + S_1)}{(4S_2 - S_1)^3} \\ \frac{\partial V_2}{\partial S_2} = & \frac{\theta^{+2}}{\theta^+ - \theta^-} \cdot \frac{(8S_2^2 - 2S_1 S_2 + S_1^2)(2S_2 - S_1)}{(4S_2 - S_1)^3} \end{cases} \tag{9}$$

Denote the vector for the first order conditions of the profits functions as $V = \begin{bmatrix} \frac{\partial V_1}{\partial S_1} \\ \frac{\partial V_2}{\partial S_2} \end{bmatrix}$, and the vector of qualities as $S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$. Then the Hessian matrix is calculated as the followings:

$$[\partial V]\,[\partial S]^{-1} = \begin{bmatrix} \frac{\partial^2 V_1}{\partial S_1^2} & \frac{\partial^2 V_2}{\partial S_1 \partial S_2} \\ \frac{\partial^2 V_1}{\partial S_1 \partial S_2} & \frac{\partial^2 V_2}{\partial S_2^2} \end{bmatrix} = \frac{\theta^{+2}}{\theta^+ - \theta^-} \begin{bmatrix} \frac{2S_2^2(8S_2 - S_1)}{(4S_2 - S_1)^4}, & \frac{8S_2(S_2 - S_1)S_1}{(4S_2 - S_1)^4} \\ -\frac{2S_1 S_2 (S_1 + 8S_2)}{(4S_2 - S_1)^4}, & \frac{8S_1^2(S_1 - S_2)}{(4S_2 - S_1)^4} \end{bmatrix} \tag{10}$$

According to the Hessian matrix and the condition $S_1 \leq S_2$, it is obviously that $\frac{\partial^2 V_1}{\partial S_1^2} > 0$ and $\frac{\partial^2 V_1}{\partial S_1 \partial S_2} < 0$. Thus, we can get the following optimal levels of the price, quantity, quality, and profits for each firm. Lemma 1 summarizes the property of the equilibrium qualities with the uniform type of the income distribution. The same results were also obtained by Bonanno (1986).

$$X_1^* = X_2^* = \frac{1}{3} \cdot \frac{\theta^+}{\theta^+ - \theta^-} \tag{11}$$

$$P_1^* = P_2^* = \frac{1}{3}\theta^+ \bar{S} \tag{12}$$

$$V_1^* = V_2^* = [\frac{1}{9} \cdot \frac{\theta^{+2}}{\theta^+ - \theta^-}]\bar{S} \tag{13}$$

LEMMA 1. *Consider a duopoly where firms set quality levels in stage one and engage in quantity competition in stage two. Assume that in equilibrium the market is uncovered, and that quality cost is zero. Then both firms will choose the highest quality level. (See the proof in Appendix)*

For this equilibrium to be consistent with the assumption of uncovered market, and with the requirement that the sum of outputs is less than the population, the ratio $\frac{\theta^+}{\theta^-}$ must be greater than three. This will be discussed in full below. Next, we will discuss under what conditions the subgame-perfect Nash equilibrium will be unique. Because the Cournot competition only exists in the uncovered market, the range of $\frac{\theta^+}{\theta^-}$ which defines the uncovered market is also the definition range of the Cournot competition. This requires that $\theta_L = \frac{P_1^*}{S_1^*} = \frac{1}{3}\theta^+ \geq \theta^-$. This restriction, together with the requirement that the sum of outputs is smaller than the population size ($X_1 + X_2$ must be less than 1), is equivalent to the condition that $\frac{\theta^+}{\theta^-} \in (3,\ +\infty)$. However, it is easy to show that in the range $\frac{\theta^+}{\theta^-} \in (3,\ 4]$, if firm 1 chooses some alternative level of quality such that $S_1 < S_1^*$, then the market may be covered. In another words, $\frac{\theta^+}{\theta^-} \in (3, +\infty)$ is only the necessary condition for the holding of the uncovered market. Furthermore, we prove that when $\frac{\theta^+}{\theta^-} \in (4, +\infty)$, then $P_1 = \theta^+ \cdot \frac{S_1 S_2}{4S_2 - S_1} \geq \theta^-$ for any $S_1 \in [0, S_2]$ (the market is uncovered under this condition). In another words, $\frac{\theta^+}{\theta^-} \in (4, +\infty)$ is the sufficient condition for the formation of the uncovered market. Firm 1 may deviate from the optimal level and then the market will be covered in the range $\frac{\theta^+}{\theta^-} \in (3,\ 4]$. We need to compare the profits obtained from the covered and uncovered market structures to decide whether firm 1 will deviate in the range $\frac{\theta^+}{\theta^-} \in (3,\ 4]$. Unfortunately, this comparison is difficulty to make because in the have proved that if the market is covered, there will be unlimited number of equilibria. (See the proof of proposition 1 in Appendix) Based on the discussion, we obtain the following proposition directly.

PROPOSITION 1. *In a duopoly market with quality-quantity competition, zero quality-cost, and the consumers are uniformly distributed in the range $[\theta^-,\ \theta^+]$, then the unique subgame-perfect Nash equilibrium exists* *in the range* $\frac{\theta^+}{\theta^-} \in (4, +\infty)$*, i.e. both firms choose the same highest level of product's quality; there are infinite Nash equilibria within the range* $\frac{\theta^+}{\theta^-} \in [0,\ 3]$*; and the number of the Nash equilibrium in the range* $\frac{\theta^+}{\theta^-} \in (3,\ 4]$ *is uncertainty. (See the proof in Appendix)*

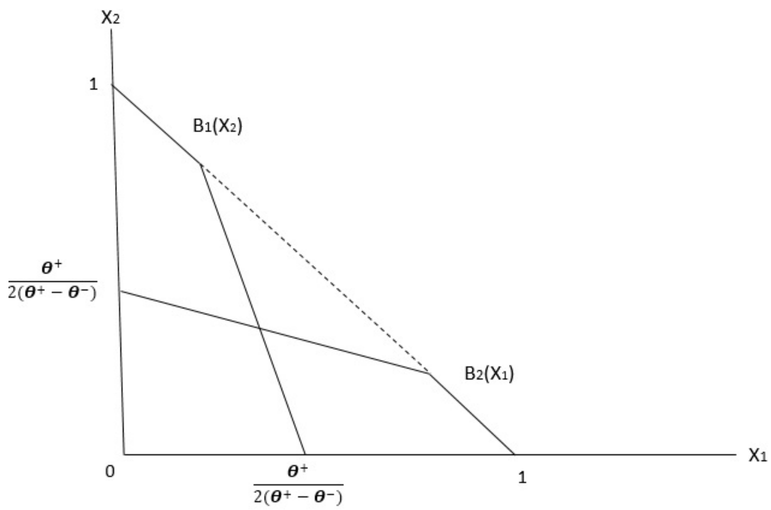

Figure 3. Quantity-strategies and the subgame-perfect Nash equilibrium

3.1.2 Results with more general type of income distribution

In this section, we will relax the uniform distribution assumption and explore the choice of quality levels by the firms in more general type of income distribution. Denote the CDF and PDF of the income distribution as $F(\theta)$ and $f(\theta)$ respectively, and assume the definition range is $\theta \in [0, \bar{\theta}]$. If we continue to assume zero quality and production cost, the theoretical analysis shows that firm 1 will always choose the same level of quality as firm 2 if the level of the income inequality is not too high. The following proposition summarizes this property.

PROPOSITION 2. *Consider a duopoly with quality-quantity competition, where higher quality does not cost more. Assume that the income distribution has the following properties:*

[1] $F(\theta)$ *invertible and twice differentiable, i.e. the inverse CDF and PDF are* $G(\Omega)$ *and* $g(\Omega)$ *respectively, and* $g'(\Omega)$ *exists;*

[2] Assume θ_m *is the mode of the distribution, i.e.* $f'(\theta) > 0$ *for* $\theta \in [0,\ \theta_m)$ *and* $f'(\theta) \leq 0$ *for* $\theta \in [\theta_m, \bar{\theta}]$*;*

[3] $[1 - F(\theta)]\frac{-f'(\theta)}{[f(\theta)]^2} \leq 2$ *for* $\theta \in [\theta_m,\ \bar{\theta}]$*;*

[4] $\frac{[1-F(\theta)]}{\theta f(\theta)} \leq 1$ *for* $\theta \in [\theta_m,\ \bar{\theta}]$.

Then both firms in the market will choose the highest level of quality, i.e. $S_1^* = S_2^* = \bar{S}$. *(See the proof in Appendix)*

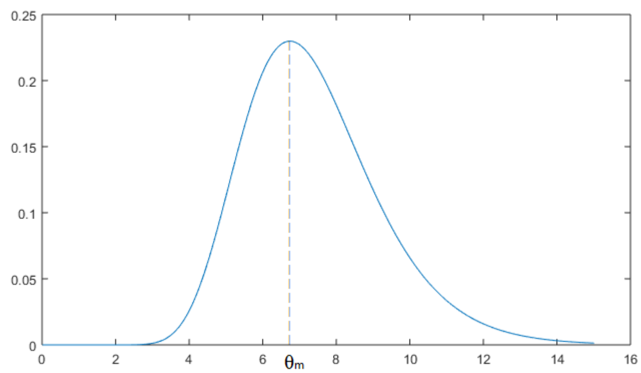

Figure 4. Example of the general form of income distribution

We check the cases of uniform, and triangular distributions in the range $\theta \in [0,1]$, and the lognormal distribution in the range $\theta \in [0,+\infty)$. We found that the uniform distribution and the symmetric triangular distribution follow the conditions [1]-[4], and the lognormal distribution $ln\mathcal{N}(\mu,\sigma^2)$ follows the conditions [1]-[4] for all $\mu = 0$ and $\sigma \leq 0.5$. In this case, we can conclude that if the income distribution changes under some specific conditions, then the quality levels by firm 1 and 2 do not change under the quality-Cournot model. These results are different from those of Yurko (2011), where the author predicts that under the quality-Bertrand model with zero cost, when the income inequality gets larger, the quality by firm 2 doesn't change but firm 1 will choose higher level of quality. Recall that proposition 2 provides only the sufficient conditions, thus we also check the numerical results with the lognormal distributions with Gini coefficients ranging from 0.15 to 0.85. When we change the inequality level, we need to exclude the effects from change of the mean of the income. In this case, we follow the Yurko (2011) and choose the pair of μ and σ to target a certain value of the Gini coefficient meanwhile keeping a constant mean of consumer's income. The computation results show that firm 1 and 2 always choose the highest level of quality no matter how the income inequality changes. Another patterns of the results are that the sales volume and price level decreasing in the income inequality. The explanation for the change of the sales and prices will be discussed in next section. *(See Figure 5 or the table 1 and 2 in Appendix)*

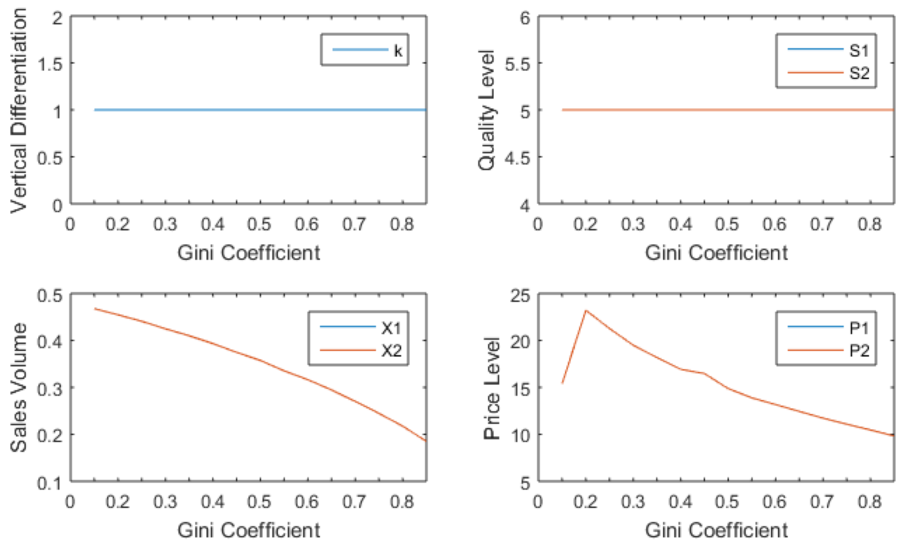

Figure 5. Results with the zero cost

3.2 Income inequality and market outcomes with convex cost function

Matto (1993) proved that with the quadratic form of quality-cost function, the firms will make vertical differentiation under the quality-Cournot model. It is difficult to obtain explicit results with the quadratic cost function while assuming a general income distribution. In this section, we try three types of the income distributions, i.e. uniform, triangular, and lognormal distributions. Firstly, we follow the Benassi, Chirco, and Colombo (2006) to explore the effects of the change of income inequality through comparing the results between the uniform distribution (high income inequality) and the triangular distribution (low income inequality). Secondly, we follow the Yurko (2011) and study the results with the lognormal distributions of different variances. In both analysis, we assume the quality-cost function in the first stage of the game as $C(S) = \frac{1}{2}S^2$. Our analysis shows the same results with the methods of Benassi, Chirco, and Colombo (2006) and Yurko (2011).

3.2.1 Results with uniform and triangular types of income distribution

The PDF and variance of the uniform and triangular distributions are described as the followings:

(1) Uniform distribution

$\theta \in [0,1]$, $f(\theta) = 1$, $\sigma^2 = \frac{1}{12}$

(2) Triangular distribution

$$\begin{cases} f(\theta) = 4\theta & \theta \in [0, \frac{1}{2}] \\ f(\theta) = 4 - 4\theta & \theta \in (\frac{1}{2}, 1] \\ \sigma^2 = \frac{1}{24} \end{cases} \tag{14}$$

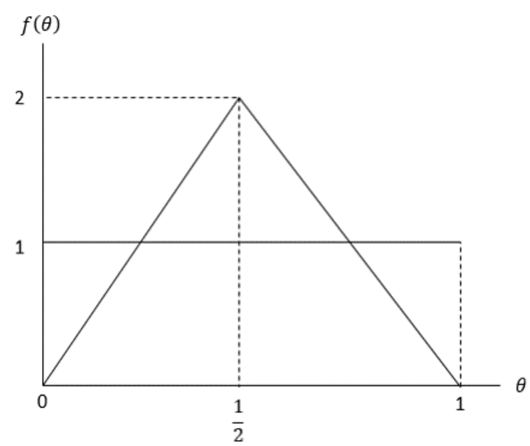

Figure 6. Uniform and triangular distributions

With the uniform income distribution, following the results of the second stage in section 3.1.1 we get the following profit functions in the second stage:

$$\begin{cases} V_1 = & \frac{S_1 S_2^2}{(4S_2 - S_1)^2} - \frac{1}{2} S_1^2 \\ V_2 = & \frac{S_2 (2S_2 - S_1)^2}{(4S_2 - S_1)^2} - \frac{1}{2} S_2^2 \end{cases} \tag{15}$$

Then solve for the equilibrium vertical differentiation and quality levels by firm 1 and 2, we get:

$$
\begin{cases}
k^* \equiv \frac{S_1^*}{S_2} & = 0.358111 \\
S_1^* = & 0.0902225 \\
S_2^* = & 0.25194
\end{cases}
\tag{16}
$$

And the sales volume by each firm is solved as:

$$
\begin{cases}
X_1^* = & 0.27 \\
X_2^* = & 0.45
\end{cases}
\tag{17}
$$

With the triangular income distribution, as discussed in the section 3.3, the possible cases of the locations of θ_L and θ_I given any values of the S_1 and S_2 include: [1]θ_L and $\theta_I \in [0, \frac{1}{2}]$; and [2] $\theta_L \in [0, \frac{1}{2}]$ and $\theta_I \in (\frac{1}{2}, 1]$. The profit functions for both firms in the second stage of the game are as followings:

$$
\begin{cases}
\pi_1 = \frac{\sqrt{2}}{2}(1 - X_1 - X_2)^{\frac{1}{2}} S_1 X_1 \\
\pi_2 = \frac{\sqrt{2}}{2}(S_2 - S_1)(1 - X_2)^{\frac{1}{2}} X_2 + \frac{\sqrt{2}}{2}(1 - X_1 - X_2)^{\frac{1}{2}} S_1 X_2 & for \;\; X_2 \geq \frac{1}{2} \\
\pi_2 = (S_2 - S_1)(1 - \frac{\sqrt{2}}{2} X_2^{\frac{1}{2}}) X_2 + \frac{\sqrt{2}}{2}(1 - X_1 - X_2)^{\frac{1}{2}} S_1 X_2 & for \;\; X_2 < \frac{1}{2}
\end{cases}
\tag{18}
$$

Take first order conditions with respect to X_1 and X_2 , we solve for the optimal X_1^* and X_2^* as:

$$
\begin{cases}
X_1^* = \frac{2}{9+2(\sqrt{3}-3)k} & for \;\; k < 0.633975 \\
X_2^* = \frac{6+2(\sqrt{3}-3)k}{9+(5\sqrt{3}-9)k} & for \;\; k < 0.633975 \\
X_1^* = \frac{2}{3}[1 - f(k)] & for \;\; k \geq 0.633975 \\
X_2^* = f(k) & for \;\; k \geq 0.633975
\end{cases}
\tag{19}
$$

where $k \equiv \frac{S_1}{S_2}$ and $f(k) \equiv \frac{\frac{2}{3}k^2}{\frac{2}{3}k^2 + [\sqrt{\frac{17-12\sqrt{2}}{8}(1-k)^2 + k^2} - \frac{4-3\sqrt{2}}{4}(1-k)]^2}$.
Then the profit functions in the first stage are:

$$
\begin{cases}
V_1 = \frac{\sqrt{2}}{2}(1 - X_1^* - X_2^*)^{\frac{1}{2}} S_1 X_1^* - \frac{1}{2} S_1^2 \\
V_2 = \frac{\sqrt{2}}{2}(S_2 - S_1)(1 - X_2^*)^{\frac{1}{2}} X_2^* + \frac{\sqrt{2}}{2}(1 - X_1^* - X_2^*)^{\frac{1}{2}} S_1 X_2^* - \frac{1}{2} S_2^2 \quad for \;\; X_2^* \geq \frac{1}{2} \\
V_2 = (S_2 - S_1)(1 - \frac{\sqrt{2}}{2} X_2^{*\frac{1}{2}}) X_2^* + \frac{\sqrt{2}}{2}(1 - X_1^* - X_2^*)^{\frac{1}{2}} S_1 X_2^* - \frac{1}{2} S_2^2 \quad for \;\; X_2^* < \frac{1}{2}
\end{cases}
\tag{20}
$$

Then we solve the equilibrium vertical differentiation and the optimal quality levels chosen by the firm 1 and 2 as:

$$
\begin{cases}
k^* \equiv \frac{S_1^*}{S_2} & = 1 \\
S_1^* = & 0.22 \\
S_2^* = & 0.22
\end{cases}
\tag{21}
$$

And then the sales of the firms are solved directly as:

$$
\begin{cases}
X_1^* = & 0.4 \\
X_2^* = & 0.4
\end{cases}
\tag{22}
$$

Comparing the results between both distributions, we found that when the income distribution gets more dispersed (higher income inequality), the vertical differentiation becomes higher, average quality level becomes lower, and the total sales volume of the firms decreases. In next section, we will show that these findings are also obtained with the variety of the lognormal distributions.

3.2.2 Results with lognormal type of income distribution

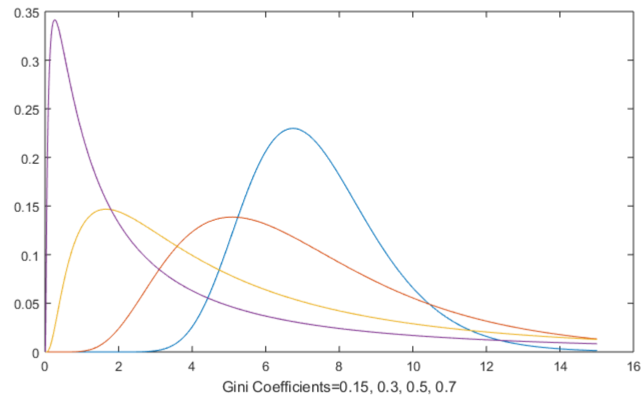

Figure 7. Lognormal distributions with different variances

As the computation in the quality-Cournot model involves the inverse distribution of the lognormal distribution, thus it is difficult to compute the results in the continuous real value range, so we assume the choice sets of $k \equiv \frac{S_1}{S_2}$, and S_2 as following: $k \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, and $S_2 \in \{0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$. For the range of the income inequality levels, we choose $\mu \in [0.75235,\ 2.04435]$,$\sigma \in [0.27,\ 1.63]$ or the Gini coefficient ranging from 0.15 to 0.75. The reason for choosing this range is that the Gini coefficients of most countries are located in this range. In addition, we continue to assume the mean of consumer's income constant. Then the equilibrium results associated with different distributions are described in Figure 8. *(Also see the Table 3 and 4 in Appendix)*

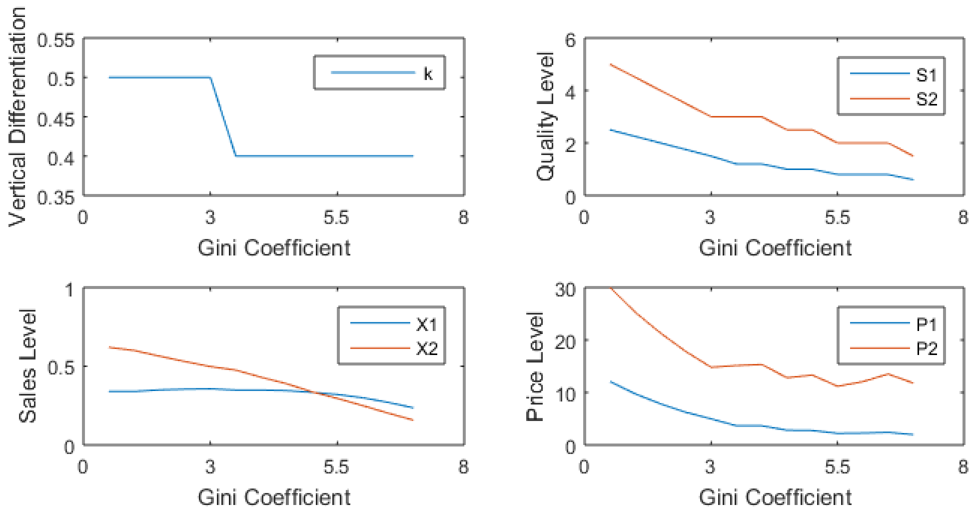

Figure 8. Results with the convex quality-cost

The figure 8 reports the results when the parameter of the cost function equals to 0.5, i.e. $C(S) = cS^2$ and $c = 0.5$. From the table above, we found two patterns: the vertical differentiation slightly increases (at least non-decreasing) in the income inequality level ; and the quality levels chosen by both firms decrease in the income inequality level. Compared with the findings under the quality-Bertrand model by Yurko (2011), there are two main differences: firstly, in Yurko (2011), the average quality level decreases in the income inequality, but under the quality-Cournot model, the average quality level increases in the income inequality; secondly, in Yurko (2011) the vertical differentiation level decreases in the income inequality, but in this paper, the vertical differentiation level increases in the income inequality when the cost function is convex and constant when the quality-cost is negligible. We also test the robustness of the results with the other values for c, i.e. $c = 0.1$ and 0.25. The results hold robust under all the cases we tested, but if the cost scale is too large, e.g. $c > 0.75$, the increase of the vertical differentiation level in the income inequality will not be significant. That's because when the cost for quality improvement is too high, the change of the marginal revenue from the variety of the income distribution will be negligible compared to the cost. In conservative sense, we could conclude that the vertical differentiation level is non-decreasing in the income inequality and significantly increasing in the latter term when the cost for quality improvement is not too high.

The reason for the differences of the results between the two studies is that the intensity of the quantity competition is quite lower than that of the price competition. When the income inequality level is low, the consumers are relatively more concentrated. Under the price competition, the concentration of the consumers would enhance the competition between the firms. Thus, the low quality firm would choose much low level of the quality to differentiate itself from the high quality firm. When the income inequality is high, the intensity of the competition will be alleviated, then the low type firm will raise up its product's quality to attract more high type consumers. Contrarily, under the quantity competition, the intensity of the competition between the firms is much lower than the price competition. Thus, the dominated factors affecting the choice of the quality levels become the cost structure of the quality-improvement and the characteristics of the income distribution. Firstly, we separate the consumers into three groups: high income group, middle income group, and low income group. From the results regarding the market separation we know that the competition between both firms takes place in the middle income group, thus the density of the consumers in the middle income group determines the market outcomes. When the income inequality level is low, the density of the middle-income consumers will be relatively high. In this case, it is easy for the firms to expand the sales volume by increasing the quality level. In another words, the firms can expand revenue with relatively low cost. However, if the income inequality is high, the density of the middle-income consumers will be low. Thus it becomes more costly to attract the consumers by improving the quality. In this case, both firms will choose to save the expenditures on the quality-improvement rather than expanding the sales.

Another pattern of the results is that the low quality firm decrease more in quality level than the high quality firm with the increasing of the income inequality. In another words, the vertical differentiation increases in the income inequality. The mechanism that leads to this result is as followings: when the income inequality increases, the sales of the both firms decrease, but at the same time the costumers of the high quality products get richer; then the price of the high quality products doesn't drop as dramatically as the low quality products; in this case, the marginal revenue of the quality-improvement for the high quality firm is higher than that for the low quality firm; and thus the high quality firm doesn't need to reduce the quality as much as the low quality firm to save the cost.

3.3 Discussion on the price, profit and number of firms

From table 4, we can see that the price and profits for both firms decrease in the income inequality. The intuition behind the results is as following. When the income inequality increases, the density of the high and low income consumers increases meanwhile the middle income class declines. As each consumer is assumed to purchase one unit of the product, the movement of the consumers from the middle income class to the high income class will not change the demand of the products, however the flows of the middle income consumers to the low income group will reduce the size of the group which could be affordable of the products. It means that some consumers will deviate from purchasing the products. If the price and quality of the firms do not change, the total market scale will inevitably shrink. In this case, the firms need to adjust their price and quality downwards in order to satisfy the shrink of the market demand.

Before discussing the equilibrium number of firms existing in the free-entry market, we need to assume that each firm needs to make the sunk entry cost. Without loss of generality and simplify our analysis, we only look at the case when the competition type turns from the duopolistic market to the monopolistic market. In our simulation case, the profit of the low type firm is always lower than that of the high type firm. In the difination range of the Gini coefficient, i.e. $Gini \in [0.15, 0.75]$, when the sunk cost is lower than 0.29, then there will be two firms in the market. If the sunk cost is between 0.29 and 0.79, then there will be two firms in the market when the income inequality is low but only one firm in the market when the income inequality is high enough. That means the number of the firms in the market decrease in the income inequality.

In next section, we will do some empirical test on our predictions about the firm's price strategy and the equilibrium number of the firms.

4 Empirics

We used the Chinese export firm data, from the years 2001 and 2006. The data set is retrieved from the replicated data set by Fan et al. (2015). We test how the products' average price and the firm number change in response to the variety of the Gini coefficients of the destination countries. The estimation formula is specified as followings.

$$P_{ijkt} = \alpha + \beta * GINI_{kt} + X_i\gamma_1 + Z_{kt}\gamma_2 + \delta_t + \eta_j + \varepsilon_{ijkt} \tag{23}$$

$$N_{jkt} = \alpha + \beta * GINI_{kt} + Z_{kt}\gamma + \delta_t + \eta_j + \varepsilon_{jkt} \tag{24}$$

The empirical results show that the average price and number of firms decrease in the income inequality.

5 Extensions

5.1 Welfare comparison under the Constant marginal quality-cost

The purpose of this section is to make some extensions of the Matto (1993), which compares the social welfare between the Cournot and Bertrand models under the quadratic quality-cost function. Matto (1993) found that with the convex cost function, the social welfare of the Bertrand model is always higher than the Cournot model. However, Matto (1993) also claimed that the welfare comparison result may be inverse under other type of the cost structure. In this section, we will check this statement with the constant marginal cost function, i.e. $C(S) = cS$, where the parameter c captures the scale of the marginal cost. To simplify the analysis, following Matto (1993), we assume the consumer's preference is uniformly distributed at $\theta \in [0,1]$. The direct computation results show that with constant marginal quality-cost function, with the cost level high enough, the Cournot model is more efficient than the Bertrand model in terms of social welfare.

With this cost structure, the direct computation results for both the Cournot and Bertrand models are as followings.

Cournot model:

$$\begin{cases} S_1^* = S_2^* = \bar{S} \\ X_1^* = X_2^* = \frac{1}{3} \\ P_1^* = P_2^* = \frac{1}{3}\bar{S} \\ V_1^* = V_2^* = [\frac{1}{9} - c]\bar{S} \end{cases} \tag{25}$$

Bertrand model:

$$\begin{cases} S_1^* = k^* S_2^* \\ S_2^* = \bar{S} \\ X_1^* = \frac{1}{4-k^*} \\ X_2^* = \frac{2}{4-k^*} \\ P_1^* = \frac{k^*(1-k^*)}{4-k^*} \\ P_2^* = \frac{2(1-k^*)}{4-k^*} \end{cases} \tag{26}$$

where k^* is solved from $\frac{4-7k^*}{(4-k^*)^3} = c$.

The social welfare is composed by the directed summation of the consumer's surplus and the firm's profits. Specifically, the equation for the social welfare is:

$$W = \frac{1}{2\delta}[\int_{\theta_L}^{\theta_I}(S_1\theta - P_1)d\theta + \int_{\theta_I}^{1}(S_2\theta - P_2)d\theta] + P_1X_1 + P_2X_2 - S_1c - S_2c \tag{27}$$

where θ_I denotes the type of consumer who is indifferent between choosing low and high quality products and θ_L denotes the lowest type of consumer who purchases the low quality products.

Denote the social welfare with the Cournot model as W^C and with Bertrand model as W^B, then it is easy to compute the relevant social welfare as:

$$\begin{cases} W^C = & (\frac{4}{9} - 2c)\bar{S} \\ W^B = & [\frac{3}{2} \cdot \frac{4-k^{*2}}{(4-k^*)^2} - (k^*+1)c]\bar{S} \end{cases} \tag{28}$$

W^B is an increasing function of k^*. When c goes to zero, k^*goes to $\frac{4}{7}$, and W^B goes to $\frac{15}{32}$ and W^C goes to $\frac{4}{9}$. As $\frac{15}{32} > \frac{4}{9}$, so if the marginal cost is small enough, then the welfare with the Bertrand model will be higher than the Cournot model. However when c goes to $\frac{1}{16}$, then k^*goes to 0, and W^Bgoes to $\frac{5}{16}$ and W^C goes to $\frac{23}{72}$. As $\frac{5}{16} < \frac{23}{72}$, so with enough high marginal cost, the welfare of the Cournot model will surpass the Bertrand model. In another paper, Hackner (2000) claimed that the if the number of firms in the market is greater than two, then prices may be higher under the price competition than the quantity competition, thus the social welfare associated with the quantity competition may be higher than the price competition under this condition. However, this statement is hard to test under the vertical differentiation model, because it is difficult to get explicit results from the quality-Cournot model with more than two firms.

5.2 Within-firm vertical differentiation

In the previous sections, we assume that one firm can only choose single level of quality and the quality differentiation is between the firms. Here, I would like to discuss the situation where the firm is allowed to choose more than one level of the quality. In another words, the quality differentiation can be made within the single firm. This is a more general assumption which is more closed to the reality. We will discuss this issue by the following steps: firstly, we discuss the case with only one firm in the market who can choose two different levels of the quality and then decide the quantity or the price of the products; secondly, we allow the two firms existing in the market and each firm can choose two levels of the quality. In the first step, as the decision is made by only one firm, thus the results will be no different between choosing the quantity or price strategy. To make the analysis more robust, we still try both the quantity and price strategies.

5.2.1 The monopolistic market with fixed cost function in the second stage

Firstly, we will discuss the behaviors of the firms under the monopolistic market. The question is whether the firm will provide various qualities under the monopolistic market. Again, without loss of generality, we assume that $S_1 \leq S_2$. In this case, if the firm invests in the quality-improvement up to the level S_2, then the cost for the firm to provide quality S_1 would be zero. This feature is different from the case with two firms in the market where both the S_1 and S_2 need the investment. The cost function of improving the quality is assumed as the convex form $C(S_1,S_2)$ with the properties such that $\frac{\partial C(S_1,S_2)}{\partial S_1} = 0$, $\frac{\partial C(S_1,S_2)}{\partial S_2} < 0$ and $\frac{\partial^2 C(S_1,S_2)}{\partial S_2^2} \geq 0$. In this case, the profits functions in the first and second stages can be written as the followings.

If the firm chooses the quantity-strategies, we have:

$$\pi = S_1X_1(1-X_1-X_2)+X_2[(1-X_2)S_2-S_1X_1]-C(S_1,S_2) \tag{29}$$

Take first order conditions, we have:

$$\begin{cases} X_1^* = & 0 \\ X_2^* = & \frac{1}{2} \end{cases} \tag{30}$$

Thus, with the quantity-strategies, the firm will only provide the high quality product.

With the price-strategies, we have:

$$\pi = P_1(\frac{P_2 - P_1}{S_2 - S_1} - \frac{P_1}{S_1}) + P_2[1 - \frac{P_2 - P_1}{S_2 - S_1}] - C(S_1, S_2) \tag{31}$$

Take first order conditions with respect to P_1 and P_2, and then take first order conditions with respect to S_1 and S_2 , we have:

$$\begin{cases} P_1^* = & \frac{1}{2}S_1 \\ P_2^* = & \frac{1}{2}S_2 \\ S_1^* = & S_2 \\ f(S_2^*) = & \frac{1}{4} \end{cases} \tag{32}$$

where $f(S_2) \equiv \frac{\partial C(S_1,S_2)}{\partial S_2}$.

As $S_1^* = S_2$, thus the firm will provide only the high quality product with the price-strategies as well.

5.2.2 The monopolistic market with the variable cost in the second stage

In this section, we will adjust the cost function from the previous section. The cost function here is assumed to be associated with both the quantity and quality of the products, i.e. $C(S,X) = c(S) \cdot X$, where X is the quantity of the products, and $c'(S) > 0$. That means the variable cost of each unit of the products is positive and also positively correlated with the quality level. This assumption can be found easily in the reality, for example, the luxury car is more expensive than the normal quality car because the components of the car are expensive. Based on this setting, we can write the profits function of the firm as followings.

Based on the quantity-strategies, we have the profits function as:

$$\pi = S_1X_1(1 - X_1 - X_2) + X_2[(1 - X_2)S_2 - S_1X_1] - c(S_1) \cdot X_1 - c(S_2) \cdot X_2 \tag{33}$$

Take first order conditions of the formula (31) with respect to X_1 and X_2 , we get:

$$\begin{cases} X_1^* = & \frac{S_1c(S_2)-S_2c(S_1)}{2S_1\cdot(S_2-S_1)} \\ X_2^* = & \frac{S_2-S_1-[c(S_2)-c(S_1)]}{2(S_2-S_1)} \end{cases} \tag{34}$$

Based on the price strategies, we have the profits function as:

$$\pi = [P_1 - c(S_1)](\frac{P_2-P_1}{S_2-S_1} - \frac{P_1}{S_1}) + [P_2 - c(S_2)](1 - \frac{P_2-P_1}{S_2-S_1}) \tag{35}$$

Take first order conditions of the formula (33) with respect to P_1 and P_2, we get:

$$\begin{cases} P_1^* = & \frac{1}{2}c(S_1) + \frac{1}{2}S_1 \\ P_2^* = & \frac{1}{2}c(S_2) + \frac{1}{2}S_2 \end{cases} \tag{36}$$

Both strategies will lead to the following profits function in the first stage:

$$\pi = \frac{[c(S_1)]^2S_2 - 2c(S_1)c(S_2)S_1 + S_1\{2S_1c(S_2) + S_2^2 - S_1S_2 - 2S_2c(S_2) + [c(S_2)]^2\}}{4S_1(S_2-S_1)} \tag{37}$$

If the cost function $c(S)$ is linear in S, then it is easy to get that $X_1^* = 0$. To obtain an explicit result with the convex cost function, we assume $c(S) = \frac{1}{2}S^2$. Based on this assumption and taking the first order conditions with respect to S_1 and S_2, we get:

$$\begin{cases} S_1^* = & 0.4 \\ S_2^* = & 0.8 \end{cases} \tag{38}$$

Thus, the firm will differentiate the quality of the products if the cost function is non-linear and increasing in both quantity and quality of the products.

5.2.3 The duopolistic competition market

In this section, we will discuss the structure of the cost function and the vertical differentiation within the single firm under the duopolistic competition market. To simplify the analysis, we assume the quality space contains only two levels of the quality, i.e. $S \in \{S_L, S_H\}$ and $S_L \leq S_H$. The cost function is increasing in both the quality level and the quantity produced, i.e. $C(S,X) = c(S) \cdot X$, where $c(S)$ is the cost on quality-improving and X is the quantity produced. That means the cost is variable to both the quality level and the quantity. Denote the cost of providing high quality $c(S_H)$ as c_H and the low quality as c_L, then providing X units of high quality products will generate the cost as $c_H \cdot X$ and low products will cost $c_L \cdot X$. Without loss of generality, we discuss the quality choice by the firm 1 given the quality level chosen by the firm 2. Generally, there are three possibilities of choices by firm 2: choosing high quality only, i.e. $S_2 = S_H$; choosing low quality only, i.e. $S_2 = S_L$; and choosing both the high and low qualities, i.e. $S_{2a} = S_H$ and $S_{2b} = S_L$. In the following part of this section, we will discuss the best strategy of firm 1 in response to the choice of firm 2 and find out the subgame-perfect Nash equilibrium. Denote X_{ia} as the quantity of the high quality products by firm i and X_{ib} is the quantity of the low quality products, where $i = 1,2$.

In the case of the Bertrand model, because the competition is too intensive in the second stage, both firms will obtain zero profits if they provide the same quality levels of the product. Thus, it is obvious that both firms will only differentiate their own products from the other competitor and will not differentiate their own products. In this case, we will only discuss the situation with the Cournot competition in detail.

(1) Firm 2 chooses high quality

In this case, the profits functions of firm 1 and firm 2 are:

$$\begin{cases} \pi_1 = & X_{1b}[S_L(1 - X_{1a} - X_{1b} - X_{2a} - c_L] + X_{1a}[S_H(1 - X_{1a} - X_{2a}) - S_L X_{1b}] - c_H X_{1a} \\ \pi_2 = & X_{2a}[S_H(1 - X_{1a} - X_{2a}) - S_L X_{1b} - c_H] \end{cases} \tag{39}$$

with constraints $X_i \geq 0$, where i = 1a, 1b, and 2a.

Take first order conditions with respect to X_{1a} , X_{1b} , and X_{2a}, we obtain the following results:

$$\begin{cases} X_{1a}^* = & max\{\ 0,\ \frac{S_H - S_L - c_H + c_L}{2(S_H - S_L)} - \frac{S_H - c_H}{6S_H}\} \\ X_{1b}^* = & max\{\ 0,\ \frac{S_L - c_L}{2S_L} - \frac{S_H - S_L - c_H + c_L}{2(S_H - S_L)}\} \\ X_{2a}^* = & max\{\ 0,\ \frac{S_H - c_H}{3S_H}\} \end{cases} \tag{40}$$

(2) Firm 2 chooses both the high and low quality

In this case, the profits functions of firm 1 and firm 2 are:

$$\begin{cases} \pi_1 = & [S_L(1-X_{1a}-X_{2a}-X_{1b}-X_{2b})-c_L]X_{1b}+[S_H(1-X_{1a}-X_{2a})-S_L(X_{1b}+X_{2b})-c_H]X_{1a} \\ \pi_2 = & [S_L(1-X_{1a}-X_{2a}-X_{1b}-X_{2b})-c_L]X_{2b}+[S_H(1-X_{1a}-X_{2a})X_{2a}-S_L(X_{1b}+X_{2b})-c_H]X_{2a} \end{cases} \tag{41}$$

with constraints $X_i \geq 0$, where i = 1a, 1b, 2a and 2b.

Take first order with respect to X_{1a}, X_{1b}, X_{2a} and X_{2b}, we obtain the following results:

$$\begin{cases} X_{1a}^* = X_{2a}^* = max\{\ 0, & \frac{1}{3}[\frac{3S_H^2-4S_LS_H-4c_HS_L+4c_LS_H}{(3S_H-4S_L)S_H}]\} \\ X_{1b}^* = X_{2b}^* = max\{\ 0, & (\frac{c_H}{2S_H}-\frac{c_L}{2S_L})\cdot\frac{2S_H}{3S_H-4S_L}\} \end{cases} \tag{42}$$

(3) Firm 2 chooses the low quality

In this case, the profits functions of firm 1 and firm 2 are:

$$\begin{cases} \pi_1 = & [S_L(1-X_{1a}-X_{1b}-X_{2b})-c_L]X_{1b}+[S_H(1-X_{1a})-S_L(X_{1b}+X_{2b})-c_H]X_{1a} \\ \pi_2 = & [S_L(1-X_{1a}-X_{1b}-X_{2b})-c_L]X_{2b} \end{cases} \tag{43}$$

Take first order with respect to X_{1a}, X_{1b}, and X_{2b}, we obtain the following results:

$$\begin{cases} X_{1a}^* = & max\{\ 0,\ \frac{1}{2}-\frac{c_H-c_L}{2(S_H-S_L)}\} \\ X_{1b}^* = & max\{\ 0,\ \frac{3S_L(c_H-c_L)-(S_H-S_L)\cdot(S_L+2c_L)}{6S_L(S_H-S_L)}\} \\ X_{2b}^* = & max\{\ 0,\ \frac{S_L+2c_L}{3S_L}\} \end{cases} \tag{44}$$

Based on the direct computation, it is easy to obtain the necessary condition for the single firm to make the within-firm vertical differentiation as $\frac{c_H}{S_H} > \frac{c_L}{S_L}$. In another words the marginal cost must be convex in the quality level. Next, we will show that if the cost of providing the high quality product is not too high and the vertical differentiation is significant enough, then there will exist a sub-perfect Nash equilibrium such that both firms will differentiate their own products.

PROPOSITION 3. *In the duopolistic model with quality-quantity competition, if the cost function is convex in quality, the variable cost of the high quality products is not too high, and the vertical differentiation is large enough, i.e.*

$$\begin{cases} \frac{c_L}{S_L} < \frac{c_H}{S_H} \\ \frac{c_H}{S_H} - \frac{c_L}{S_L} < \frac{3S_H - 4S_L}{4S_L} \\ \frac{S_L}{S_H} < \frac{3}{4} \\ \frac{c_H - c_L}{S_H - S_L} < 1 \end{cases}$$

Then there exists a subgame-perfect Nash equilibrium in the market such that both firms will provide both the high and low quality products. (See the proof in Appendix)

The intuition of the relevant results is that the firm needs to consider both the cost and the substitute effects when deciding the provision of the product's quality. The cost effect means that because the marginal cost is convex in the quality level, thus the average cost in terms of both the quality level and the quantity will be higher with the high quality level than the low quality level. In this case, the firm will save much cost if providing the low quality products rather than the high quality products. The substitute effect means that if the firm provides the low quality products, some consumers who purchase the high quality products when the low quality products are absent in the market will turn to buy the low quality products. In another words, some consumers will be attracted from the high quality market to the low quality market. Because the marginal revenue from the high quality product is higher than that from the low quality product, the show-up of the low quality products will reduce the total revenue. The firms decide the quantity of the low and high quality products by considering both the substitute and the cost effects.

6 Conclusion

The paper explores the changes of the market outcomes in response to a variety of income distribution under the quality-Cournot model . Our study found that when the quality-cost is zero and the income inequality is not too high, then both firms will choose the highest quality level. If the quality-cost function is convex, the average quality level will decrease and the vertical differentiation level will increase. in the income inequality. These results are different from those of the Yurko (2011), which assumed the quality-Bertrand competition and reached the conclusion that with both the zero and convex quality-cost, the average quality level always increases and the vertical differentiation level always decreases in the income inequality. The intuition of this difference is that under the price competition, the competition is much intensive and thus the competition effects dominates the cost effects. Contrarily, under the quantity competition case the competition is much moderate. Thus the firms would consider more about the location of the consumers (market shares) and the benefits from lowering the quality-cost. The paper also discusses the conditions for the single firm to choose multiple levels of the product's quality. The analytical results show that the necessary conditions for the firms to make within-firm vertical differentiation include: the vertical differentiation between the high and low qualities is large enough; the marginal production cost is small enough; and the quality-cost function is convex.

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Appendix

Proof of the lemma 1:

Define functions $v_1(S_1) \equiv \frac{\partial V_1}{\partial S_1}$ and $v_2(S_2) \equiv \frac{\partial V_2}{\partial S_2}$. As $\frac{\partial^2 V_1}{\partial S_1^2} > 0$ and the minimum value of $v_1(S_1)$ is $v_{1min} = v_1(0) = \frac{1}{32}\frac{(\mu+\delta)^2}{\delta} \geq \frac{1}{8}\mu \geq 0$, so $v_1(S_1) > 0$ for all $S_1 \in [0, S_2]$. Thus, the maximum value of V_1 is obtained at $S_1^* = S_2$. Given the best strategy of firm 1, $v_2(S_2) > 0$ for any $S_2 \in [0, \bar{S}]$. Thus the highest value of V_2 is obtained at $S_2 = \bar{S}$. In this case, both firms will entry the market and choose the same highest level of the quality. More detailed proof can be found in Bonanno (1986).

Q.E.D.

Proof of the proposition 1:

As the discussion above, when $\frac{\theta^+}{\theta^-} \in (4, +\infty)$, the unique perfect Nash equilibrium exists, X_1^* and $X_2^*$are obtained as (11) to (14) with the constraints $X_1 + X_2 < 1$. And it is obviously that no firm will deviate from the equilibrium with choosing an alternative pair $(S_i,\ X_i)$, for $i = 1, 2$. More detailed proof can be found in Bonanno (1986) and Motta (1993).

Now we focus the situation when $\frac{\theta^+}{\theta^-} \in [0,\ 3]$ and $\frac{\theta^+}{\theta^-} \in (3,\ 4]$. As $P_1 \geq S_1\theta^-$, that is equivalent to have the constraint $X_1 + X_2 \leq 1$. Without loss of generality, we continue to have the assumption $S_1 \leq S_2$. In the second stage of the competition the profits functions of firm 1 and 2 are $\pi_1 = S_1X_1[\theta^+ - (\theta^+ - \theta^-)\cdot(X_1 + X_2)]$ and $\pi_2 = X_2[(\theta^+ - (\theta^+ - \theta^-)\cdot X_2)S_2 - (\theta^+ - \theta^-)\cdot S_1X_1]$ respectively. With the constraint $X_1 + X_2 \leq 1$, we can get the following quantity strategies for firm 1 and 2.

$$\begin{cases} X_1 = \frac{\theta^+}{2(\theta^+ - \theta^-)} - \frac{1}{2}X_2 & if\ \ \frac{\theta^+}{2(\theta^+ - \theta^-)} + \frac{1}{2}X_2 < 1 \\ X_1 = 1 - X_2 & otherwise \end{cases} \tag{45}$$

$$\begin{cases} X_2 = \frac{\theta^+}{2(\theta^+ - \theta^-)} - \frac{S_1}{2S_2}X_1 & if\ \ \frac{\theta^+}{2(\theta^+ - \theta^-)} + \frac{2S_2 - S_1}{2S_2}X_1 < 1 \\ X_2 = 1 - X_1 & otherwise \end{cases} \tag{46}$$

If we substitute the first equation of (44) into the first equation of (43), we can get the following result $X_1 + X_2 = \frac{\theta^+}{\theta^+ - \theta^-}(\frac{3S_2 - S_1}{4S_2 - S_1})$. With the range$\frac{\theta^+}{\theta^-} \in [0,\ 3]$ and the condition $0 < S_1 \leq S_2$(both firms entry the market), this result is greater than 1, i.e. $X_1 + X_2 > 1$, violating the constraint that $X_1 + X_2 \leq 1$. Thus it is obviously that both firms will choose the strategies $X_1^* = 1 - X_2$ if $X_2 \geq \frac{\theta^+ - 2\theta^-}{\theta^+ - \theta^-}$ and $X_2^* = 1 - X_1$ if $X_1 \geq \frac{S_2}{2S_2 - S_1} \cdot \frac{\theta^+ - 2\theta^-}{\theta^+ - \theta^-}$ respectively. In this case, if $\frac{S_2}{2S_2 - S_1} \cdot \frac{\theta^+ - 2\theta^-}{4\theta^+ - \theta^-} < \frac{\theta^-}{\theta^+ - \theta^-}$, there will be unlimited number of equilibria, i.e. $(X_1^*, X_2^*) \subseteq \{(X_1, X_2) \mid X_1 + X_2 = 1,\ X_1 \in [\frac{S_2}{2S_2 - S_1} \cdot \frac{\theta^+ - 2\theta^-}{\theta^+ - \theta^-},\ \frac{\theta^-}{\theta^+ - \theta^-}], X_2 \in [\frac{\theta^+ - 2\theta^-}{\theta^+ - \theta^-},\ \frac{2S_2\theta^+ - 2(\theta^+ - \theta^-)S_1}{2(\theta^+ - \theta^-)\cdot(2S_2 - S_1)}]\}$. It is easy to see that in the range $\frac{\theta^+}{\theta^-} \in (3,\ 4]$, if firm 1 chooses relative high level of S_1 the Nash equilibrium is unique, and if firm 1 choose relative low level of S_1 the number of Nash equilibria is uncountable. Thus the payoffs of both firms are uncertain if firm 1 chooses relative low level of S_1. In this case, we cannot figure out whether firm 1 will choose high or low level of quality. Then the number of Nash equilibria in the second stage will be uncertain.

Q.E.D.

Proof the proposition 2:

Denote the area $\theta \in [0, \theta_m)$ as I, and the area $\theta \in [\theta_m, \bar{\theta}]$ as II. Denote the lowest type of consumer who chooses the low quality product as θ_L and the consumer who is indifferent between choosing high and low quality product as θ_I, then we have the following three possibilities of the locations of θ_L and θ_I: (1) $\theta_{L,}\ \theta_I \in I$; (2) $\theta_L \in I,\ \theta_I \in II$; and (3) $\theta_{L,}\ \theta_I \in II$. In the rest part of content, we will look at the quality choices by firms under these three conditions.

[1] $\theta_{L,}\ \theta_I \in I$

The profit function of firm 1 in the second stage is $\pi_1 = S_1 X_1 G(1 - X_1 - X_2)$. Then take the first order condition with respect to X_1 we get $\frac{\partial \pi_1}{\partial X_1} = S_1 G(1 - X_1 - X_2) - S_1 X_1 g(1 - X_1 - X_2) = 0$, and the second order condition is $\frac{\partial^2 \pi_1}{\partial X_1^2} = -2S_1 g(1 - X_1 - X_2) + S_1 X_1 g'(1 - X_1 - X_2) < 0$, thus the best strategy of firm 1 in response to the action of firm 2 as $X_1^* = \frac{G(1 - X_1^* - X_2)}{g(1 - X_1^* - X_2)}$. As $g'(\Omega) < 0$ when $\Omega \in [0, F(\theta_m)]$, thus when X_2 increases, then X_1^* must decrease, i.e. $-1 < \frac{\partial X_1^*}{\partial X_2} < 0$. The profits of firm 2 is $\pi_2 = X_2[(S_2 - S_1)G(1 - X_2) + S_1 G(1 - X_1 - X_2)]$. Take first order condition with respect to X_2, we get $\frac{\partial \pi_2}{\partial X_2} = (S_2 - S_1)[G(1 - X_2) - X_2 g(1 - X_2)] + S_1[G(1 - X_1 - X_2) - X_2 g(1 - X_1 - X_2)] = 0$ and the second order condition $\frac{\partial^2 \pi_2}{\partial X_2^2} = (S_2 - S_1)[X_2 g'(1 - X_2) - 2g(1 - X_2)] + S_1[X_2 g'(1 - X_1 - X_2) - 2g(1 - X_1 - X_2)] < 0$. Thus the optimal choice of firm 2 is $X_2^* = \frac{S_1[G(1 - X_1 - X_2^*) - G(1 - X_2^*)] + S_2 G(1 - X_2^*)}{S_1[g(1 - X_1 - X_2^*) - g(1 - X_2^*)] + S_2 g(1 - X_2^*)}$. As $-1 < \frac{\partial X_1^*}{\partial X_2} < 0$, so it is easy to get that $\frac{\partial X_2^*}{\partial S_1} < 0$. When S_1 increases, the value of the right hand side of the equation decreases, because $G(1 - X_1 - X_2^*) - G(1 - X_2^*) < 0$ and $g(1 - X_1 - X_2^*) - g(1 - X_2^*) > 0$. The only way to make the equality holds again is to decrease the value of X_2. Thus, in the first stage of the game, we have $\frac{\partial \pi_1}{\partial S_1} = -g(1 - X_1^* - X_2^*) \frac{\partial X_2^*}{\partial S_1} S_1 X_1^* + G(1 - X_1^* - X_2^*) X_2^* > 0$. In this case, $S_1^* = S_2$.

[2] $\theta_L \in I,\ \theta_I \in II$

As $\pi_2 = X_2[(S_2 - S_1)G(1 - X_2) + S_1 G(1 - X_1 - X_2)]$, so we take first order condition with respect to X_2, then get $\frac{\partial \pi_2}{\partial X_2} = (S_2 - S_1)[G(1-X_2) - X_2 g(1-X_2)] + S_1[G(1-X_1-X_2) - X_2 g(1-X_1-X_2)] = 0$ and the second order condition $\frac{\partial^2 \pi_2}{\partial X_2^2} = (S_2 - S_1)[X_2 g'(1-X_2) - 2g(1-X_2)] + S_1[X_2 g'(1-X_1-X_2) - 2g(1-X_1-X_2)] < 0$ under the condition $[1-F(\theta)]\frac{-f'(\theta)}{[f(\theta)]^2} \leq 2$ for $\theta \in [\theta_m, \bar{\theta}]$. Set $H(\cdot) = (\frac{1}{k} - 1)[G(1-X_2) - g(1-X_2)X_2] + [G(1-X_1-X_2) - g(1-X_1-X_2)X_2] = 0$, where $k \equiv \frac{S_1}{S_2}$ is the strategy of firm 1. It is easy to get that $\frac{\partial H(.)}{\partial k} = -\frac{1}{k^2}[G(1-X_2) - g(1-X_2)X_2]$ and $\frac{\partial H(.)}{\partial X_2} = -g(1-X_1-X_2) + [g'(1-X_1-X_2)X_2 - g(1-X_1-X_2)](\frac{\partial X_1^*}{\partial X_2} + 1) - (\frac{1}{k} - 1)[2g(1-X_2) - X_2 g'(1-X_2)]$. As $[1-F(\theta)]\frac{-f'(\theta)}{[f(\theta)]^2} \leq 2$ for $\theta \in [\theta_m, \bar{\theta}]$ and $\frac{[1-F(\theta)]}{\theta f(\theta)} \leq 1$ for $\theta \in [\theta_m, \bar{\theta}]$, so it is easy to see that both $\frac{\partial H(.)}{\partial k} < 0$, $\frac{\partial H(.)}{\partial X_2} < 0$. Thus $\frac{\partial X_2}{\partial k} = -[\frac{\partial H(.)}{\partial k}]/[\frac{\partial H(.)}{\partial X_2}] < 0$. As $\pi_1 = S_1 X_1 G(1-X_1-X_2) = kS_2 X_1 G(1-X_1-X_2)$, so $\frac{\partial \pi_1}{\partial k} = S_2 X_1 G(1-X_1-X_2) - kS_2 X_1 g(1-X_1-X_2)\frac{\partial X_2}{\partial k} > 0$. In this case the best choice of firm 1 is such that $k^* = 1$.

[3] $\theta_{L,}\ \theta_I \in II$

In this case, $\frac{\partial \pi_1}{\partial X_1} = S_1 G(1-X_1-X_2) - S_1 X_1 g(1-X_1-X_2) > 0$ because $\frac{[1-F(\theta)]}{\theta f(\theta)} \leq 1$ for $\theta \in [\theta_m, \bar{\theta}]$. Then the game goes back to the case [2], and then the best choice of firm 1 is such that $k^* = 1$.

Q.E.D.

Proof of the proposition 3:

With the conditions listed in the proposition 3, the best responses of firm 1 under cases (1), (2), and (3) are choosing both high and low quality, choosing both high and low quality, and choosing only high quality respectively. Thus, no matter what strategies of firm 2 chooses, choosing low quality only is dominated strategy for firm 1. Because firm 1 and firm 2 are symmetric, both firms will not choose the strategy that choosing the low quality only. Given the potential strategies chosen by the opponent, the best response for both firms is choosing both the high and low quality.

Q.E.D.

Gini	k^*	S_1^*	S_2^*	$\tilde{S}$
0.15	1	5	5	5
0.2	1	5	5	5
0.25	1	5	5	5
0.3	1	5	5	5
0.35	1	5	5	5
0.4	1	5	5	5
0.45	1	5	5	5
0.5	1	5	5	5
0.55	1	5	5	5
0.6	1	5	5	5
0.65	1	5	5	5
0.7	1	5	5	5
0.75	1	5	5	5
0.8	1	5	5	5
0.85	1	5	5	5

Table 1. The quality with the zero cost

Gini	P_1^*	P_2^*	X_1^*	X_2^*	π_1	π_2
0.15	15.4	15.4	0.47	0.47	7.20	7.20
0.2	23.2	23.2	0.45	0.45	10.55	10.55
0.25	21.3	21.3	0.44	0.44	9.37	9.37
0.3	19.5	19.5	0.42	0.42	8.28	8.28
0.35	18.2	18.2	0.41	0.41	7.45	7.45
0.4	16.9	16.9	0.39	0.39	6.65	6.65
0.45	16.5	16.5	0.37	0.37	6.17	6.17
0.5	14.9	14.9	0.36	0.36	5.32	5.32
0.55	13.9	13.9	0.34	0.34	4.65	4.65
0.6	13.2	13.2	0.32	0.32	4.17	4.17
0.65	12.4	12.4	0.29	0.29	3.67	3.67
0.7	11.7	11.7	0.27	0.27	3.17	3.17
0.75	11.1	11.1	0.24	0.24	2.71	2.71
0.8	10.5	10.5	0.22	0.22	2.28	2.28
0.85	9.8	9.8	0.19	0.19	1.82	1.82

Table 2. The price and quantity with zero cost

Gini	k^*	S_1^*	S_2^*	$\tilde{S}$
0.15	0.5	2.5	5	3.75
0.2	0.5	2.25	4.5	3.375
0.25	0.5	2	4	3
0.3	0.5	1.75	3.5	2.625
0.35	0.5	1.5	3	2.25
0.4	0.4	1.2	3	2.1
0.45	0.4	1.2	3	2.1
0.5	0.4	1	2.5	1.75
0.55	0.4	1	2.5	1.75
0.6	0.4	0.8	2	1.4
0.65	0.4	0.8	2	1.4
0.7	0.4	0.8	2	1.4
0.75	0.4	0.6	1.5	1.05

Table 3. The quality with the convex cost function

Gini	P_1^*	P_2^*	X_1^*	X_2^*	π_1	π_2
0.15	12.1	30	0.34	0.62	0.99	6.10
0.2	9.7	25.3	0.34	0.6	0.78	5.04
0.25	7.8	21.3	0.35	0.56	0.74	4.01
0.3	6.2	17.8	0.35	0.53	0.68	3.31
0.35	5.0	14.8	0.36	0.50	0.65	2.87
0.4	3.7	15.1	0.35	0.48	0.56	2.70
0.45	3.7	15.3	0.35	0.43	0.56	2.09
0.5	2.8	12.8	0.35	0.39	0.47	1.86
0.55	2.8	13.3	0.33	0.34	0.42	1.37
0.6	2.2	11.2	0.32	0.30	0.39	1.31
0.65	2.3	12.1	0.30	0.25	0.36	1.02
0.7	2.4	13.5	0.27	0.20	0.33	0.73
0.75	2.0	11.8	0.24	0.16	0.29	0.73

Table 4. The price and quantity with the convex cost function

VARIABLES	(1) $log(Price)$	(2) $log(Price)\_average$	(3) $log(Firm\_number)$
$GINI$	-0.00767***	-0.00854***	-0.00748***
	(0.000490)	(0.000700)	(0.000416)
$tariff$	0.153**	0.0819	-1.130***
	(0.0707)	(0.103)	(0.0615)
$log(CPI)$	0.0261***	0.0180**	-0.109***
	(0.00561)	(0.00794)	(0.00473)
$log(GDP)$	0.0307***	0.100***	0.250***
	(0.00264)	(0.00408)	(0.00243)
$log(GDP\_pc)$	-0.000874	0.0114	-0.0824***
	(0.00789)	(0.0108)	(0.00641)
TFP	-0.0402***		
	(0.00670)		
HHI	0.132*		
	(0.0780)		
$log(labor)$	-0.102***		
	(0.00678)		
$log(sales)$	0.0947***		
	(0.00760)		
$capital\_labor\_ratio$	0.0124***		
	(0.00367)		
$log(wage)$	0.272***		
	(0.00702)		
$Constant$	-1.176	-2.593**	-4.829***
	(0.827)	(1.043)	(0.620)
Observations	149,229	44,229	44,229
R-squared	0.723	0.833	0.523
Time FE	YES	YES	YES
Industry FE	YES	YES	YES

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4. Price, firm number and the income inequality of the destination countries